Conference on Real Reductive Groups and Theta Correspondence

Tianyuan Mathematics Research Center

July 21-27, 2024

Schedule

All the talks are scheduled in Beijing time (GMT+8).

Monday, July 22

Time	Speaker	Title	Host
8:50-9:00	Opening		
9:00-10:00	Wee Teck Gan	The Relative Langlands Duality	Binyong Sun
10:30-11:30	David Vogan	What does the unitary dual look like?	
11:30-13:30	Lunch Break		
13:30-14:30	Dan Barbasch	Dirac cohomology for complex reductive groups	Lei Zhang
15:00-16:00	Kayue Daniel Wong	Some progress on the unitary dual of $U(p,q)$	
16:30-17:30	Jia-Jun Ma	Local Theta Correspondence and Its Applications	

Tuesday, July 23

Time	Speaker	Title	Host
9:00-10:00	Jun Yu	Casselman's comparison conjecture	Angela Pasquale
10:30-11:30	Tomasz Przebinda	Symmetry Breaking Operators for Dual Pairs with One Member Compact	
11:30-13:30	Lunch Break		
13:30-14:30	Atsushi Ichino	Nearly holomorphic modular forms and congruences	Hung Yean Loke
15:00-16:00	Rui Chen	Homological aspects of theta correspondence	
16:30-17:30	Gordan Savin	Exceptional dual pairs where one member is $SL(3)$	

Wednesday, July 24

Time	Speaker	Title	Host
9:00-10:00	Dougal Davis (online)	Mixed Hodge modules and unipotent representations	Shilin Yu
10:30-11:30	Yongchang Zhu	Siegel-Weil Formula and Average of Certain Conformal Field Theories	
11:30-13:30	Lunch Break		
13:30-17:30	Free Discussion		

Thursday, July 25

Time	Speaker	Title	Host
9:00-10:00	Shuichiro Takeda	MVW-involutions on p-adic groups	Anne-Marie Aubert
10:30-11:30	Kei Yuen Chan	Algorithms for computing parabolic inductions and Jacquet modules for GL over local fields	
11:30-13:30	Lunch Break		
13:30-14:30	Kyo Nishiyama	Double flag variety over reals	Hiroyuki Ochiai
15:00-16:00	Chen Wan	Some examples of the relative Langlands duality	
16:30-17:30	Dmitry Gourevitch (online)	A Stone-von Neumann equivalence of categories for smooth representations of the Heisenberg group	

Friday, July 26

Time	Speaker	Title	Host
9:00-10:00	Dihua Jiang (online)	Branching Problem, Local Descent, and Arithmetic Wavefront Set	Jia-Jun Ma
10:30-11:30	Jing-Song Huang	Decomposing Tensor Product by Dirac Cohomology	
11:30-13:30	Lunch Break		
13:30-17:30	Free Discussion		

Title and Abstract

Dirac cohomology for complex reductive groups

Dan Barbasch (Cornell University)

Dirac cohomology is an invariant which generalizes $(\mathfrak{g}, K)$-cohomology and captures interesting properties of unitary representations. Some time ago, Pandzic and myself made a precise conjecture that classifies unitary representations with nontrivial Dirac cohomology for complex reductive groups. This talk will review the background and previous work, and in particular report on joint work with Daniel Wong that completes this classification.

Algorithms for computing parabolic inductions and Jacquet modules for GL over local fields

Kei Yuen Chan (The University of Hong Kong)

Let F be a local field. Parabolic inductions and Jacquet functors are basic tools in the representation theory to construct representations. In particular, the Langlands classification is to build irreducible representations from parabolic inductions. In another instance, the celebrated multiplicity one theorem of Sun-Zhu has some consequences on the structure of Jacquet modules.

In this talk, we consider the problem of computing simple quotients of parabolic inductions and Jacquet functors in both real and p-adic general linear groups. We shall explain some algorithms in computing those invariants. This is based on a joint work with Daniel Wong and an ongoing work with Basudev Pattanayak.

Homological aspects of theta correspondence

Rui Chen (Zhejiang University)

Following Adams–Prasad–Savin, one can study some homological version of the theta correspondence. In this talk we show two Ext-vanishing results for the theta correspondence, and give applications to Ext-analogues of branching laws. Besides, we will exhibit some interesting examples that Ext spaces are non-zero. These are partially based on a joint work with Jialiang Zou.

Mixed Hodge modules and unipotent representations

Dougal Davis (University of Edinburgh)

It is a long-standing expectation that the unitary dual of a real reductive Lie group is built from an as-yet-undefined set of building blocks, the unipotent representations. Whatever the eventual definition, the annihilators of unipotent representations are expected to be the unipotent ideals recently constructed by Losev, Mason-Brown and Matvieievskyi by quantising covers of nilpotent co-adjoint orbits. I will discuss joint work in preparation with Lucas Mason-Brown, in which we prove that, when the boundary of the orbit is not too large, every irreducible representation annihilated by a unipotent ideal is (a) a quantisation of a vector bundle on a K-orbit

and (b) unitary. Our proof, which is uniform across all cases, uses the Hodge-theoretic characterisation of unitarity proved recently by myself and Vilonen. Our results cover all unipotent representations of complex groups (in particular, special unipotents), including exceptional groups.

The Relative Langlands Duality

Wee Teck Gan (National University of Singapore)

I will give an introduction to the conjectural relative Langlands duality, as envisioned in the recent work of Ben-Zvi, Sakellaridis and Venkatesh. Then I will discuss my joint work with Bryan Wang demonstrating instances of the purported duality. The key ingredient used in our work is a result of Raul Gomez and Chengbo Zhu on the behaviour of generalized Whittaker models under the theta correspondence.

A Stone-von Neumann equivalence of categories for smooth representations of the Heisenberg group

Dmitry Gourevitch (Weizmann Institute of Science)

I would like to give a virtual talk on: arXiv:2201.12638. Alternatively, I can also talk on: arXiv:2109.00204 Again.

Decomposing Tensor Product by Dirac Cohomology

Jing-Song Huang (The Chinese University of Hong Kong, Shenzhen)

Let $\mathfrak{g}$ be a complex semisimple Lie algebra. Let F be a finite-dimensional $\mathfrak{g}$ module with weights $\mu_1, \cdots, \mu_k$, and X be an arbitrary $\mathfrak{g}$-module with infinitesimal character χ_λ. Kostant proved that an infinitesimal character which occurs in the tensor product of $X \otimes F$ is necessarily of the form $\chi_{\lambda+\mu_i} (i = 1, \cdots, k)$. However, it is a difficult question whether a non-zero submodule with infinitesimal character $\chi_{\lambda+\mu_i}$ indeed occurs in $X \otimes F$. Assume X is a Harish-Chandra module with infinitesimal character λ. We prove a criterion when a nonzero submodule with infinitesimal character $\chi_{\lambda+\mu_i}$ occurs in $X \otimes F$ by using Dirac cohomology.

Nearly holomorphic modular forms and congruences

Atsushi Ichino (Kyoto University)

This is a report on work in progress with Kartik Prasanna. We consider an analog of the Bertolini-Darmon-Prasanna formula for $GSp(4)$ which relates the special value of the p-adic L-function with the p-adic Abel-Jacobi image of some cycle. In the process, we need to construct explicitly a solution of some p-adic differential equation, in which nearly holomorphic modular forms play an important role. We explain this construction, with emphasis on representation theory of real reductive groups. We also discuss some observation on congruences between K-types, which is necessary to modify a solution to a "p-adically holomorphic" one.

Branching Problem, Local Descent, and Arithmetic Wavefront Set

Dihua Jiang (University of Minnesota)

As a special case of the classical branching problem, we discuss our recent work of the explicit spectral decomposition of the local descents, which are refinements of the local Langlands correspondence and the local Gan-Gross-Prasad conjecture for classical groups over any local field of characteristic zero. Taking those local descents as the first step, we develop the method of L-parameter descent and introduce the notion of Arithmetic Wavefront Sets. The talk is based on the joint work with C. Chen, D. Liu and L. Zhang.

Local Theta Correspondence and Its Applications

Jia-Jun Ma (Xiamen University)

In the 1970s, Roger Howe initiated the systematic study of theta correspondence. Today, theta is undoubtedly one of the most powerful methods in the representation theory of reductive groups. This presentation will review some theorems and tools that emerged in studying theta correspondence, such as the degenerate principal series, conservation relation, and moment maps. We will then explain how these tools can be applied to study certain basic representations, such as unipotent and cuspidal representations. In the end, I will discuss a geometric approach revealing the relationship between theta correspondence and Springer theory.

Double flag variety over reals

Kyo Nishiyama (Aoyama Gakuin University)

Let H be a symmetric group of a real reductive algebraic group G. For a parabolic subgroups Q of H and P of G, we call $X = H/Q \times G/P$ a double flag variety. I will talk on the finiteness of H-orbits on X and a classification of orbits.

Symmetry Breaking Operators for Dual Pairs with One Member Compact

Tomasz Przebinda (University of Oklahoma)

Given a locally compact topological group A, a closed subgroup $B \subseteq A$, a continuous representation α of A on a linear topological space and an irreducible representation β of B, the space $\mathrm{Hom}_B(\alpha, \beta)$ of the intertwining maps is known as the space of the symmetry braking operators. This is a vector space which may have an additional structure. If $C \subseteq A$ is a subgroup commuting with B, then $\mathrm{Hom}_B(\alpha, \beta)$ is a representation of C. If α is realized in a space of functions on a manifold then $\mathrm{Hom}_B(\alpha, \beta)$ may consists of differential or pseudodifferential operators.

In this talk $A = \widetilde{Sp}(W)$ is the metaplectic group, $\alpha = \omega$ is the Weil representation, $B = \widetilde{G}\widetilde{G'}$, where (G, G') is real reductive dual pair in the symplectic group $Sp(W)$ and $\beta = \Pi \otimes \Pi'$ is the irreducible representation of $\widetilde{G}\widetilde{G'}$ which occurs as a quotient of the space of the smooth vectors of ω. In this case the space of the symmetry breaking operators is one dimensional. Hence, up to a non-zero complex multiple there is only one symmetry breaking operator.

We realize ω in a Schrödinger model on the Hilbert space $L^2(X)$, where $X \subseteq W$ is a maximal isotropic subspace. We focus on the case when the group G is compact. Then the symmetry breaking operator is not a differential oparator but a rather singular pseudodifferential operator. Its Weyl symbol is a distribution on the symplectic space W. We compute this distribution explicitly in terms of the orbital integrals on the symplectic space.

By specifying further to the case when G' is compact, we recover by purely analytic methods the existence of Howe correspondence for compact unitary groups, or equivalently, Weyl's First Fundament Theorem of the Classical Invariant Theory for complex general linear groups.

This is joint work with Mark McKee and Angela Pasquale.

Exceptional dual pairs where one member is $SL(3)$

Gordan Savin (University of Utah)

For groups over p-adic fields, we consider a family of dual pairs $(SL(3), G)$ in exceptional groups, where G is the group of automorphisms of a cubic space (J, N). Here J is a rank 3 Jordan algebra, and N the cubic norm on J. We use ping-pong of periods to prove that exceptional theta correspondences are one to one. As a consequence, we can analyze a family of rank 2 symmetric spaces G/H where H is the stabilizer of a generic point in J. For example, in E_8, we have the dual pair $(SL(3), E_6)$ and the rank two symmetric space is E_6/F_4. This is a joint work with P. Bakic and W. T. Gan.

MVW-involutions on p-adic groups

Shuichiro Takeda (Osaka University)

In this talk, I will talk on the notion of MVW-involution on p-adic groups and known examples of MVW-involutions. Also discussed are the general Spin and Pin groups and their MVW-involutions, which is a joint work with M. Emory.

What does the unitary dual look like?

David Vogan (Massachusetts Institute of Technology)

Gelfand in the 1930s introduced the problem of finding the "unitary dual" of any locally compact group G. I'll talk about this problem in case G is a reductive Lie group.

If K is a compact connected Lie group, then Elie Cartan and Hermann Weyl gave an elegant and powerful description of the unitary dual almost a hundred years ago.

Since that time, this unitary dual problem has been solved for more and more complex examples; but almost never has the answer been formulated so well as in the work of Cartan and Weyl.

I will formulate a conjecture (the "FPP conjecture") about the answer for a general real reductive G. This conjecture originates in the work of Dan Barbasch in the late 1980s; a great deal of supporting evidence can be found in the work of Barbasch and his collaborators since that time. The formulation I will give is joint work with Jeffrey Adams, Stephen Miller, and Marc van Leeuwen.

The FPP conjecture defines for each real reductive G a finite collection of compact polyhedra $FPP_j(G)$, each of dimension at most the semisimple rank of G. The entire unitary dual is (conjecturally) a countable union of pieces, each isomorphic to some $FPP_k(L)$ for an appropriate Levi subgroup L of G.

For $SL(2,\mathbb{R})$ there are exactly six of these polyhedra: five points (the first two discrete series, the two limits of discrete series, and the reducible unitary principal series) and one interval $[0,1]$ (the spherical complementary series).

The atlas software is able in principle to calculate all the polyhedra $FPP_j(G)$. For the split real form of E_7, the number of polyhedra is 2025524; computing all of them requires approximately a month of CPU time, using a few hundred gigabytes of memory.

For the split form of E_8, the number of polyhedra is about 300 million. Computing them with current technology will require using hundreds or thousands of CPUs at once, each with access to several hundred gigabytes of memory. We hope to accomplish this in the next year or so.

In order for this work to constitute a reasonable answer to the title question, we need to UNDERSTAND the polyhedra $FPP_j(G)$. Dan Barbasch's work in the 1980s, and his subsequent collaborations, offered such an understanding in some special cases, beginning from Arthur's description of unitary representations conjecturally relevant to automorphic forms. We have not seriously begun to extend his work.

Some examples of the relative Langlands duality

Chen Wan (Rutgers University – Newark)

In this talk I will discuss some examples of the relative Langlands duality (introduced by Ben-Zvi—Sakellaridis—Venkatesh) for strongly tempered spherical varieties. In some cases, I will introduce a relative trace formula comparison for the models and prove the fundamental lemma/smooth transfer. This is a joint work with Zhengyu Mao and Lei Zhang.

Some progress on the unitary dual of $U(p,q)$

Kayue Daniel Wong (The Chinese University of Hong Kong, Shenzhen)

A central yet unsolved problem in representation theory of reductive groups is to classify all irreducible, unitary representations (the unitary dual). In this talk, we will mention some recent findings on the unitary dual of $U(p,q)$. If time permits, we will also discuss how these results can be generalized to other real reductive groups.

This talk is based on joint work with Dan Barbasch and Hongfeng Zhang.

Casselman's comparison conjecture

Jun Yu (Peking University)

Casselman's comparison conjecture concerns a close relation between the nilpotent Lie algebra homology of a Harish-Chandra module and its Casselman-Wallach globalization. We show a proof in the case of taking homology with respect to the nil-radical of a minimal real parabolic subalgebra. This is a joint work with Ning Li and Gang Liu.

Siegel-Weil Formula and Average of Certain Conformal Field Theories

Yongchang Zhu (The Hong Kong University of Science and Technology)

In this talk, we will discuss recent applications of Siegel-Weil formula in mathematical physics. It is known that the arithemetic quotient

$$M = O_{n,n}(\mathbb{Z})\backslash O_{n,n}(\mathbb{R})/O_n(\mathbb{R}) \times O_n(\mathbb{R})$$

appears in the physical literature as the Narain moduli space of two-dimensional conformal field theory obtained by toroidal compactitication in n-dimension. The genus one partition function for $m \in M$ is given by the the Siegel theta series $\theta(m, \tau)$, where τ is in the upper half plane parametrizing the genus one Riemann surface. A. Maloney and E. Witten proposed that the average of the theory is linked to certain simple theories of gravity in three dimensions. And they computed the average partition function $\int_M \theta(m, \tau) dm$ using the Siegel-Weil formula. In the later part of the talk, we discuss the computation of the average of correlations functions, aiming to provide generalizations of the Siegel-Weil formula, and and elucidate their physical implications.